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Constraints for the existence of flat and stable non-supersymmetric vacua in supergravity

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Abstract: We further develop on the study of the conditions for the existence of locally stable non-supersymmetric vacua with vanishing cosmological constant in supergravity models involving only chiral superfields. Starting from the two necessary conditions for flatness and stability derived in a previous paper (which involve the Kähler metric and its Riemann tensor contracted with the supersymmetry breaking auxiliary fields) we show that the implications of these constraints can be worked out exactly not only for factorizable scalar manifolds, but also for symmetric coset manifolds. In both cases, the conditions imply a strong restriction on the Kähler geometry and constrain the vector of auxiliary fields defining the Goldstino direction to lie in a certain cone. We then apply these results to the various homogeneous coset manifolds spanned by the moduli and untwisted matter fields arising in string compactifications, and discuss their implications. Finally, we also discuss what can be said for completely arbitrary scalar manifolds, and derive in this more general case some explicit but weaker restrictions on the Kähler geometry.

Keywords: Supergravity Models, Supersymmetry Breaking.

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Contents

1. Introduction

Supergravity theories represent one of the phenomenologically most promising class of supersymmetric extensions of the standard model [1, 2]. They are also very well motivated at the theoretical level, since they can emerge as low-energy effective theories of string models. Furthermore, the moduli fields arising in string compactifications to four dimensions seem to be natural candidates to constitute the hidden sector that is supposed to be responsible for supersymmetry breaking [3, 4]. From a phenomenological point of view, this type of models must however posses some characteristics in order to be viable: supersymmetry must be broken, the cosmological constant should be tiny, and all the extra moduli fields of the hidden sector should be stabilized with a sufficiently large mass. In the low energy effective theory all these crucial features are controlled by a single quantity, the four-dimensional scalar potential, which gives information on the dynamics of the moduli fields, on how supersymmetry is broken and on the value of the cosmological constant. The characterization of the conditions under which a supersymmetry-breaking stationary point of the scalar potential satisfies simultaneously the flatness condition (vanishing of the cosmological constant) and the stability condition (the stationary point is indeed a minimum) is therefore very relevant in the search of phenomenologically viable string models.

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Recently, some substantial progress has been achieved in the search of these non-supersymmetric Minkowski/dS vacua in the context of string/M-theory compactifications. This was mainly related to the understanding of the superpotentials generated by background fluxes [5] and by non-perturbative effects like gaugino condensation [6], which suggested then new interesting possibilities for model building, like in particular those proposed in refs. [7, 8]. On the other hand, it is interesting to note that the structure of the Kähler potential is usually fixed by the symmetries of the compactification, whereas the form of the superpotential is more difficult to characterize. Therefore any information allowing for a discrimination among compactifications with different Kähler potentials, independently of the form of the superpotential, is extremely valuable. One could then hope to eventually combine all these sources of information to try to identify a hopefully restricted set of phenomenologically viable models.

In this respect it was shown by the authors in ref. [9] that, in models involving an arbitrary number of chiral multiplets but no vector multiplets, it is possible to derive two very simple and strong conditions for the existence of flat and stable vacua, which are necessary but in general not sufficient. These conditions involve the Kähler metric and its Riemann tensor as well as the vector of auxiliary fields controlling the direction of supersymmetry breaking. One can then imagine a situation with a fixed Kähler potential and an arbitrary superpotential and study the space of solutions admitted by these constraints by scanning over all the possible values of the vector of auxiliary fields satisfying the restrictions with a fixed Kähler metric and Riemann tensor. This constrains the Kähler geometry of the scalar manifold. Explicit expressions for these constraints were derived in [9] in the case where the Kähler manifold spanned by the scalar fields is a factorizable space. In that case, the resulting conditions restrict the Kähler curvature scalars associated to each of the scalar fields and the ratios of the supersymmetry-breaking auxiliary fields defining the Goldstino direction. These results were then applied to the dynamics of moduli fields arising in some string compactifications.

The aim of this paper is to further develop on the study initiated in [9] and analyze in more generality the implications that the flatness and stability constraints have on theories with more general scalar manifolds. We will study in detail the class of theories where the scalar manifold is not factorizable but has instead the special feature of being symmetric, as is the case for instance for the coset spaces that are relevant for string models. Actually in this case of symmetric manifolds it is possible to derive, as in the case of factorizable manifolds, very simple and strong constraints. We will also study what can be said in the general case where the scalar manifold is completely arbitrary.

The paper is organized as follows: In section 2 we review the derivation of the necessary conditions for flatness and stability of ref. [9], which represent the starting point of our analysis. In section 3 we examine what kind of implications can be extracted from these constraints for completely general scalar manifolds. In sections 4 and 5 we study instead the special cases of factorizable and symmetric scalar manifolds, where more specific information restricting the Kähler geometry can be obtained and we also derive the bounds on the values that the auxiliary fields can take. In section 6 we apply our results to the particularly interesting case of homogeneous coset spaces arising in the moduli sector of

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string compactifications. In section 7 we also comment on the implications of our results for supersymmetric Randall-Sundrum models. Finally in section 8 we summarize our results.

2. Conditions for flat and stable vacua

The Lagrangian of a minimal four-dimensional supergravity theory with N chiral superfields is entirely specified, at the leading two-derivative order, by a single arbitrary real function G depending on the corresponding chiral superfields Φ_i and their conjugates $\Phi_i^\dagger$, as well as on its derivatives [10]. The function G is the following Kähler invariant combination of the real Kähler potential K and the holomorphic superpotential W (we use Planck units $M_{\rm P} = 1$):

$$G(\Phi_i, \Phi_i^\dagger) = K(\Phi_i, \Phi_i^\dagger) + \log W(\Phi_i) + \log \bar{W}(\Phi_i^\dagger)\,. \tag{2.1}$$

Mixed holomorphic and antiholomorphic derivatives of G depend only on the Kähler potential K and define the Kähler geometry of the manifold parametrized by the scalar fields, whose metric is given by $g_{i\bar{\jmath}} = G_{i\bar{\jmath}}$. Purely holomorphic or antiholomorphic derivatives of G depend instead also on the superpotential W, and control the way supersymmetry is broken. The auxiliary fields F^i of the chiral multiplets are determined by their equations of motion to have the values $F^i = e^{G/2} G^i$. The potential for the scalar fields ϕ^i takes then the following simple form:

$$V = e^G \left(G^i G_i - 3 \right)\,. \tag{2.2}$$

In order to study the existence of non-supersymmetric Minkowski minima in a potential of the type (2.2) one should first check under which conditions this potential has a stationary point with vanishing cosmological constant. The flatness condition of vanishing cosmological constant implies that $V = 0$ at the minimum, implying:

$$g_{i\bar{\jmath}} G^i G^{\bar{\jmath}} = 3\,. \tag{2.3}$$

The stationarity condition implies instead that $\nabla_i V = 0$ at the vacuum, which leads to the following equations:

$$G_i + G^k \nabla_i G_k = 0\,, \tag{2.4}$$

where ∇_i denotes the covariant derivative on the Kähler manifold, which when applied to G_i gives $\nabla_i G_k = g_{ik} - G_{ij\bar{l}} G^{\bar{l}}$.

Finally, to ensure the stability of the stationary point, one should check that the matrix of second derivatives of the potential is positive definite.[1] This matrix can be also computed using covariant derivatives, since the extra connection terms vanish by the flatness and stationarity conditions. There are two different n-dimensional blocks, $V_{i\bar{\jmath}} = \nabla_i \nabla_{\bar{\jmath}} V$ and $V_{ij} = \nabla_i \nabla_j V$, and after a straightforward computation (see also ref. [12, 13]) these are

[1] Notice that we require that the cosmological constant should be tunable to zero order by order in perturbation theory, and that there should not be any flat direction. Models of the no-scale type [11] are thus excluded from our analysis.

found to be given by the following expressions:

$$\begin{aligned} V_{i\bar{j}} &= e^G \left(g_{i\bar{j}} + \nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}p\bar{q}}\, G^p\, G^{\bar{q}} \right), \\ V_{ij} &= e^G \left(\nabla_i G_j + \nabla_j G_i + \frac{1}{2} G^k \{\nabla_i, \nabla_j\} G_k \right), \end{aligned} \tag{2.5}$$

where $R_{i\bar{j}p\bar{q}}$ denotes the Riemann tensor on the Kähler manifold, whose components are given by $R_{i\bar{j}p\bar{q}} = G_{i\bar{j}p\bar{q}} - g^{k\bar{l}}\, G_{ip\bar{l}}\, G_{\bar{j}\bar{q}k}$.

The full $2n$-dimensional matrix of second derivatives at the stationary point has the form:

$$V_{IJ} = \begin{pmatrix} V_{i\bar{j}} & V_{ij} \\ V_{\bar{i}\bar{j}} & V_{\bar{i}j} \end{pmatrix} . \tag{2.6}$$

The conditions under which this $2N$-dimensional matrix is positive definite are difficult to work out in general [2], the only way being to study in full detail the behavior of all the $2N$ eigenvalues. However, it was shown in [9] that it is possible to deduce some simple necessary conditions for the matrix (2.6) to be positive definite. This is done by using the property that if a matrix is positive definite then all its upper-left submatrices are also positive definite. This implies, for instance, that the N-dimensional submatrix $V_{i\bar{j}}$ should be positive definite. In particular along the direction in the scalar field space defined by G^i [3] one must therefore have $V_{i\bar{j}} G^i \bar{G}^{\bar{j}} > 0$. It is straightforward to show that, using the flatness and stationarity conditions as well as the results (2.5), this leads to the extremely simple necessary conditions for stability [4]:

$$R_{i\bar{j}p\bar{q}}\, G^i G^{\bar{j}} G^p G^{\bar{q}} < 6 \, . \tag{2.7}$$

Equations (2.3) and (2.7) represent simple and very strong constraints that should be fulfilled by any theory for the existence of non-supersymmetric vacua that are flat and stable. It is important to realize that the metric $g_{i\bar{j}}$ and the curvature tensor $R_{i\bar{j}p\bar{q}}$ depend only on the Kähler potential and therefore on the geometry. On the other hand, the quantities G^i depend also on the superpotential and define the way in which supersymmetry is broken, since they are related to the auxiliary fields by the relation $G^i = F^i/m_{3/2}$, where $m_{3/2} = e^{G/2}$. One can then imagine a situation with a fixed Kähler potential and an arbitrary superpotential. More precisely, one can treat $g_{i\bar{j}}$ and $R_{i\bar{j}p\bar{q}}$ as fixed quantities and scan over all the possible values of G^i satisfying the restriction (2.3) and the bound (2.7). It is then clear that eq. (2.7) puts constraints on the values that the various G^i can take, and actually requiring eq. (2.7) to have a solution also requires that $g_{i\bar{j}}$ and $R_{i\bar{j}p\bar{q}}$ satisfy certain conditions.

Notice that the two conditions (2.3) and (2.7) are evaluated at a specific stationary point, determined by the equations (2.4). It is then very convenient to switch to normal

[2] See ref. [14] for an attempt in this direction for string models with fluxes.

[3] Notice that in the fermion field space the direction defined by G^i identifies the would-be Goldstino that is absorbed by the gravitino field through the super-Higgs mechanism when supersymmetry is broken.

[4] This condition can be easily generalized to the case in which a positive cosmological constant is required, with $V = e^G\, \epsilon$ at the vacuum. One finds in that case $R_{i\bar{j}p\bar{q}}\, G^i G^{\bar{j}} G^p G^{\bar{q}} < 6 + 2\epsilon$.

coordinates around the point under consideration. This can be done by introducing the vielbein e_i^J and its inverse e_I^j, defined in the usual way to diagonalize the metric and its inverse: $g_{i\bar{j}} = e_i^R e_{\bar{j}}^{\bar{S}} \delta_{R\bar{S}}$, and $g^{i\bar{j}} = e_R^i e_{\bar{S}}^{\bar{j}} \delta^{R\bar{S}}$ (in what follows we will use capital letters to denote flat indices). The flatness and stability conditions (2.3) and (2.7) can then be rewritten in terms of the metric $\delta_{I\bar{J}} = e_I^r e_{\bar{J}}^{\bar{s}} g_{r\bar{s}}$ and the Riemann tensor $R_{I\bar{J}P\bar{Q}} = e_I^r e_{\bar{J}}^{\bar{s}} e_P^t e_{\bar{Q}}^{\bar{u}} R_{r\bar{s}t\bar{u}}$ in these coordinate, and in terms of the corresponding new variables $G^I = e_r^I G^r$:

$$\begin{aligned} &\delta_{I\bar{J}} G^I G^{\bar{J}} = 3\,, \\ &R_{I\bar{J}P\bar{Q}}\, G^I G^{\bar{J}} G^P G^{\bar{Q}} < 6\,. \end{aligned} \tag{2.8}$$

These expressions will be our starting point. They clearly represent a constraint on the curvature evaluated at the given stationary point and on the direction of supersymmetry breaking. The strength and the simplicity of these constraint depend on the type of scalar manifold. In particular, it is clear that for homogeneous manifolds with constant curvature they will translate into very direct constraints on the parameters of the theory.

Unfortunately, as the conditions (2.8) are quadratic and quartic in the variables G^i, is not possible in general to solve such conditions exactly. To derive explicit results one must either further simplify the conditions to get a new set of weaker but still necessary conditions that can be solved exactly, or consider special types of geometries for which the problem simplifies from a complicated quartic problem to an exactly solvable quadratic problem. We will explore what can be said using both different options in the following sections.

3. General scalar manifolds

In this section we want to explore the possibility to use the conditions (2.8) to find a restriction on the Kähler geometry in a totally generic case where the manifold spanned by the scalar fields is an arbitrary Kähler manifold. In order to do that the key is to try to reduce the problem from a quartic one to a quadratic one by defining new variables that are quadratic in the N complex quantities G^i. In doing so we will unavoidably introduce new constraints, which keep the difficulty of the problem intact. However, we can then take the option of discarding the new constraints and solving the weaker set of conditions exactly to obtain a general but weaker necessary condition. There are actually two different possible ways to do this, which lead to different conditions.

A first possibility that one can consider to try to solve the conditions (2.8) is to introduce the N-dimensional positive definite Hermitian matrix of variables

$$H^{I\bar{J}} = \frac{1}{3} G^I G^{\bar{J}}\,. \tag{3.1}$$

Clearly, this does not represent a regular change of variables in terms of the G^I's. Indeed, the N^2 real components of $H^{I\bar{J}}$ are subject to the following quadratic constraints:

$$H^{I\bar{J}} H^{P\bar{Q}} = H^{I\bar{Q}} H^{P\bar{J}}\,. \tag{3.2}$$

These represent $(N-1)^2$ independent real constraints, leaving $2N-1$ real independent variables, which correspond to the N absolute values and the $N-1$ relative phases of the variables G^I. The flatness and stability conditions (2.8) can then be rewritten in terms of the new variables (3.1) as

$$\delta_{I\bar{J}}\, H^{I\bar{J}} = 1\ , \quad R_{I\bar{J}P\bar{Q}}\, H^{I\bar{J}} H^{P\bar{Q}} < \frac{2}{3}\,. \tag{3.3}$$

This is a constrained minimization problem which can be solved in the standard way using Lagrange multipliers. The difficulty now is that, in addition to the linear flatness condition, the variables $H^{I\bar{J}}$ are also subject to the quadratic constraints (3.2). The treatment of these constraints with Lagrange multipliers implies cubic terms in the functional to be minimized, and therefore the problem cannot be solved exactly. Then, as we already mentioned, the best we can do is to discard the constraints (3.2) and consider the weaker set of conditions defined by (3.3) on the variables $H^{I\bar{J}}$.

The problem defined by the eqs. (3.3) can be solved by considering the linear map $H^{I\bar{J}} \to R^{\bar{J}I}{}_{P\bar{Q}}\, H^{P\bar{Q}}$ on Hermitian tensors. This map acts on the N^2-dimensional vector space of independent components of the Hermitian tensors $H^{I\bar{J}}$, and can be represented by a $N^2 \times N^2$ matrix. This matrix can then be diagonalized, and this defines N^2 eigenvalues R_h, with $h = 1, 2, \ldots, N^2$. The corresponding eigenvectors $H_h^{I\bar{J}}$ satisfy the eigenvalue equations

$$R^{\bar{J}I}{}_{P\bar{Q}}\, H_h^{P\bar{Q}} = R_h\, H_h^{I\bar{J}}\,. \tag{3.4}$$

It is then clear from (3.4) that if any of the eigenvalues R_h are negative or vanishing, then the constraints (3.3) admit solutions as long as the variables $H^{I\bar{J}}$ are aligned closely enough to the particular directions associated to the negative or vanishing eigenvalues. On the other hand, if all the eigenvalues R_h are positive, then the constraints (3.3) can be minimized straightforwardly using Lagrange multipliers. One finds that they admit solutions only if the following bound is satisfied:

$$\delta^{I\bar{J}}\delta^{P\bar{Q}}\, R^{-1}_{I\bar{J}P\bar{Q}} > \frac{3}{2}\,. \tag{3.5}$$

A second possibility that one can consider to try to solve the conditions (2.8) is to introduce the N-dimensional complex symmetric matrix of variables

$$S^{IJ} = \frac{1}{3}\, G^I G^J\,. \tag{3.6}$$

Again, this does not represent a regular change of variables with respect to the variables G^I. Indeed, the $N(N+1)/2$ complex components of S^{IJ} are subject to the following quadratic constraints:

$$S^{IJ} S^{PQ} = S^{IQ}\, S^{PJ}\,. \tag{3.7}$$

These represent $N(N-1)/2$ independent complex constraints, so that one is left with N complex independent variables, which are in one to one correspondence with the N complex variables G^I.

It is clear that, in order to be able to use efficiently this new set of variables, we need to squared the flatness condition. By doing so, the two flatness and stability conditions (2.8) can be rewritten as:

$$\delta_{I\bar{J}}\,\delta_{P\bar{Q}}\,S^{IP}S^{\bar{J}\bar{Q}}=1\;,\quad R_{I\bar{J}P\bar{Q}}\,S^{IP}S^{\bar{J}\bar{Q}}<\frac{2}{3}\,. \tag{3.8}$$

This is again a constrained minimization problem that can be faced using Lagrange multipliers. But again there is the difficulty that, in addition to the flatness constraint, the variables S^{IJ} are also subject to the quadratic constraints (3.7). As before, the implementation of these constraints with Lagrange multipliers implies cubic terms in the functional to be minimized, and therefore the problem cannot be solved exactly either. Once again the best we can do is to discard the constraints (3.7) and consider the weaker constraints represented only by the conditions (3.8).

The problem defined by the constraints (3.8) can be solved by considering the linear map $S^{IJ}\to R_P{}^I{}_Q{}^J\,S^{PQ}$ on complex symmetric tensors. This map acts on the $N(N+1)/2$-dimensional vector space of independent components of the complex symmetric tensors S^{IJ}, and can be represented by a $N(N+1)/2\times N(N+1)/2$ matrix. This matrix can then be diagonalized, and this defines $N(N+1)/2$ eigenvalues R_s, with $s=1,2,\ldots,N(N+1)/2$. The corresponding eigenvectors $S_s{}^{IJ}$ satisfy the eigenvalue equations

$$R_P{}^I{}_Q{}^J\,S_s^{PQ}=R_s\,S_s^{IJ}\,. \tag{3.9}$$

They can be chosen to form an orthonormal and complete basis of the vector space, with $S_s{}^{IJ}S_{s'\,JI}=\delta_{ss'}$ and $\sum_s S_s{}^{IJ}S_{s\,PQ}=\delta^I_P\,\delta^J_Q$. The Riemann matrix and its inverse, whenever it exists, can then be written in the form $R_P{}^I{}_Q{}^J=\sum_s R_s\,S_s^{IJ}S_{s\,PQ}$ and $R^{-1}{}_P{}^I{}_Q{}^J=\sum_s R_s^{-1}S_s^{IJ}S_{s\,PQ}$, and the new variables S^{IJ} can be decomposed as $S^{IJ}=\sum_s S_s\,V_s^{IJ}$. Using this, the conditions (3.8) can finally be rewritten as

$$\textstyle\sum_s S_s^2=1\;,\quad \sum_s R_s\,S_s^2<\frac{2}{3}\,. \tag{3.10}$$

It is clear that these constraints admit solutions if any of the eigenvalues R_s is negative or vanishes. If instead all the eigenvalues R_s are positive, then from (3.10) we get that they have to fulfill the bound:

$$\min\{R_s\}<\frac{2}{3}\,. \tag{3.11}$$

This condition can also be rewritten as:

$$\max\Big\{\text{eigenvalues}\,\big(R^{-1}{}_P{}^I{}_Q{}^J\big)\Big\}>\frac{3}{2}\,. \tag{3.12}$$

The inequalities (3.5) and (3.12) represent two different constraints on the Kähler curvature that have to be necessarily satisfied in order for the theory to have the chance of admitting flat and stable non-supersymmetric vacua. They are valid in full generality for any supergravity theory, with an arbitrary scalar manifold, under the sole assumption that the effects due to vector multiplets can be neglected. However, as already mentioned, they contain less information than the original constraints (2.8). To derive stronger conditions

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from (2.8), without any loss of information, it is necessary to consider more specific classes of scalar manifolds where a more explicit knowledge of the constraints on the variables $H^{I\bar{J}}$ and/or S^{IJ} is available. This depends obviously on the details of the model, and therefore to get such an information one should perform a case by case analysis. We will however see in the following two sections that stronger constraints emerging directly from (2.8) can be derived for the two classes of scalar manifolds that are respectively factorizable and symmetric.

4. Factorizable scalar manifolds

A first situation in which the conditions (2.8) can be solved exactly is when the scalar manifold is factorizable into a product of one-dimensional submanifolds associated to each of the fields (this case was already worked out in detail in [9] but for completeness we will briefly review it here). For factorizable spaces the Kähler potential is separable into a sum of terms, each of them depending on a single chiral field. The Kähler metric becomes then diagonal and has only N non-zero elements $g_{i\bar{i}}$. The Riemann tensor is also completely diagonal, and has only N non-vanishing components $R_{i\bar{i}i\bar{i}}$, which are related to the diagonal components of the metric through the curvature scalars R_i of the one-dimensional submanifolds associated to each of the fields. In flat indices, one finds then that the Riemann tensor is given by:

$$R_{I\bar{J}P\bar{Q}} = \begin{cases} R_i \ , & \text{if } I = J = P = Q \, , \\ 0 \ , & \text{otherwise.} \end{cases} \ . \tag{4.1}$$

This form of the Riemann tensor implies that both maps on Hermitian and symmetric tensors introduced in the previous section have vanishing eigenvalues. More precisely, the map on Hermitian tensors has N non-vanishing eigenvalues given by the curvature scalars R_i, and $N(N-1)$ vanishing eigenvalues, so that the condition (3.3) is trivially satisfied. Similarly, the map on symmetric tensors has N non-vanishing eigenvalues given by the curvature scalars R_i, and $N(N-1)/2$ vanishing eigenvalues, so the condition (3.8) is also trivially satisfied.

Nevertheless, as was shown in [9], in this case it is possible to derive explicit results directly from the constraints (2.8), thanks to the particularly simple form (4.1) that the Riemann tensor takes, and get more restrictive necessary conditions than the one implied by (3.5) and (3.12). To do so, one needs to introduce the following N real and positive variables parametrizing the Goldstino direction:

$$\Theta_i = \frac{1}{\sqrt{3}} \left| G^I \right| . \tag{4.2}$$

The two constraints (2.8) become then

$$\textstyle\sum_i \Theta_i^2 = 1 \ , \ \ \sum_i R_i \, \Theta_i^4 < \frac{2}{3} \ . \tag{4.3}$$

These can have solutions only if some of the curvature scalars are negative or vanish, or if they are all positive but satisfy the following bound:

$$\sum_i R_i^{-1} > \frac{3}{2}\,. \tag{4.4}$$

If the restriction (4.4) is satisfied then solutions exist, but only for a limited range of values for the variables Θ_i. More precisely, the allowed interval is $\Theta_i \in [\Theta_i^-, \Theta_i^+]$, where:

$$\Theta_i^+ = \begin{cases} \sqrt{\dfrac{R_i^{-1} + \sqrt{\dfrac{2}{3} R_i^{-1}\left(\sum_{k\neq i} R_k^{-1}\right)\left(\sum_k R_k^{-1} - \dfrac{3}{2}\right)}}{\left(\sum_k R_k^{-1}\right)}}\,, & \text{if } R_i^{-1} < \dfrac{3}{2}\,, \\ 1\,, & \text{if } R_i^{-1} > \dfrac{3}{2}\,. \end{cases}$$

$$\Theta_i^- = \begin{cases} \sqrt{\dfrac{R_i^{-1} - \sqrt{\dfrac{2}{3} R_i^{-1}\left(\sum_{k\neq i} R_k^{-1}\right)\left(\sum_k R_k^{-1} - \dfrac{3}{2}\right)}}{\left(\sum_k R_k^{-1}\right)}}\,, & \text{if } \sum_{k\neq i} R_k^{-1} < \dfrac{3}{2}\,, \\ 0\,, & \text{if } \sum_{k\neq i} R_k^{-1} > \dfrac{3}{2}\,. \end{cases} \tag{4.5}$$

This also constrains the values that the auxiliary fields can take, since these are given by $|F^I| = \sqrt{3}\,\Theta_i\, m_{3/2}$.

5. Symmetric scalar manifolds

Another interesting and relevant case where one can solve the original constraints (2.8) exactly, is when the Kähler manifold spanned by the scalar fields is a coset group manifold of the form G/H, where G is the global isometry group and H the local stability group. In this case the Kähler potential K has a very special form due to the fact that the Kähler manifold has a large number of Killing vectors. These coset Kähler manifolds have been classified, and there exist finitely many types of them for each given dimensionality N (see for example [15]). All of them are Einstein manifolds and, moreover, the metric and the Riemann tensor are invariant under the global symmetry transformations of the group G, and their various components are strongly constrained. This simplifies the problem sufficiently much to enable us to solve it exactly. In addition to this fact, these spaces turn out to have constant curvature, since they are homogeneous. This suggests that the constraints emerging from the flatness and stability conditions will translate into particularly simple restrictions on the parameters of the theory. More precisely, the Riemann tensor in flat coordinates is a constant tensor that can be written in terms of a G-invariant combination of Kronecker δ-functions that are invariant under the subgroup H:

$$R_{I\bar{J}P\bar{Q}} = \text{combination of } \delta\text{-functions}\,. \tag{5.1}$$

The maps on Hermitian and symmetric tensors defined in section 2 can be easily diagonalized in this case. Indeed, the eigentensors of these maps must correspond to irreducible representations of the group H, and can be obtained by decomposing the tensors $H^{I\bar{J}}$ and S^{IJ} under $G \to H$.

All the existing Kähler coset manifolds can be studied with the same technique. The form of the Kähler potential and the Riemann tensor with flat indices for all these spaces can be found in ref. [15]. Here we shall however restrict to those few spaces that are directly relevant for the simplest string models.

5.1 $\frac{\mathrm{SU}(1,q+1)}{\mathrm{U}(1)\times\mathrm{SU}(q+1)}$

The simplest class of Kähler coset manifold is the maximally symmetric space of dimension $N = q + 1$, with the structure:

$$\mathcal{M} = \frac{\mathrm{SU}(1, q+1)}{\mathrm{U}(1) \times \mathrm{SU}(q+1)} \,. \tag{5.2}$$

This manifold is the Kählerian analogue of the sphere in Riemannian geometry. It can be parametrized by using a vector of complex fields ϕ_i, where $i = 1, 2, \ldots, q+1$, and the Kähler potential is given by

$$K = -\frac{2}{R_{\rm all}} \ln\left(1 - \textstyle\sum_i \Phi_i \Phi_i^\dagger\right) . \tag{5.3}$$

The Riemann tensor is in this case given by a tensor product of metrics, and in flat coordinates it has the simple form

$$R_{I\bar{J}P\bar{Q}} = \frac{R_{\rm all}}{2}\Big(\delta_{I\bar{J}}\,\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}}\Big) . \tag{5.4}$$

Let us now see what kind of information we can get by applying the general conditions (3.5) and (3.12) derived in section 3. The map $H^{I\bar{J}} \to R^{\bar{J}I}{}_{P\bar{Q}}\, H^{P\bar{Q}}$ on Hermitian tensors does not have any vanishing eigenvalue, and can therefore be inverted. One finds $R^{-1\,\bar{J}I}{}_{P\bar{Q}} = (2/R_{\rm all})(\delta^I_P\delta^{\bar{J}}_{\bar{Q}} - (N-1)^{-1}\delta^{I\bar{J}}\delta_{P\bar{Q}})$ and therefore the curvature constraint (3.5) implies that the overall curvature constant should satisfy $R_{\rm all} < 4/3\,(q+1)/(q+2)$. On the other hand, the map $S^{IJ} \to R_P{}^I{}_Q{}^J\, S^{PQ}$ on symmetric tensors has eigenvalue $R_{\rm all}$ with degeneracy $(q+1)(q+2)/2$, so that the condition (3.12) implies the stronger bound $R_{\rm all} < 2/3$.

In this case, however, it is also possible to solve exactly the equations (2.8), and compare it with the general conditions (3.5) and (3.12). To solve directly (2.8), we define the new positive and real variable

$$\Theta = \frac{1}{\sqrt{3}}\,\sqrt{\textstyle\sum_I |G^I|^2} . \tag{5.5}$$

The the two conditions (2.8) can then be written as

$$\Theta^2 = 1 \,, \quad R_{\rm all}\,\Theta^4 < \frac{2}{3} . \tag{5.6}$$

The situation is therefore identical to the one arising in a one-dimensional Kähler manifold with curvature $R_{\rm all}$. The constraint for the existence of non-supersymmetric flat and stable vacua is then simply

$$R_{\rm all}^{-1} > \frac{3}{2}\,. \tag{5.7}$$

When this is satisfied, there is a unique solution corresponding to $\Theta = 1$. Note that we get the same result as the one obtained by using the, in principle, less restrictive condition (3.12). This illustrates the fact that it is possible to get useful information out of the conditions (3.5) and (3.12).

5.2 $\frac{\mathrm{SU}(p,p+q)}{\mathrm{U}(1)\times\mathrm{SU}(p)\times\mathrm{SU}(p+q)}$

The next-to-simplest case of Kähler coset manifold is the Grassmanian space of dimension $N = p(p+q)$ given by:

$$\mathcal{M} = \frac{\mathrm{SU}(p,p+q)}{\mathrm{U}(1)\times\mathrm{SU}(p)\times\mathrm{SU}(p+q)}\,. \tag{5.8}$$

This is a natural and less symmetric generalization of the previous case, which is recovered for $p = 1$. It can be parametrized by a matrix of complex fields ϕ_{ia}, where $i = 1, 2, \ldots, p$ and $a = 1, 2, \ldots, p+q$. The Kähler potential is given by

$$K = -\frac{2}{R_{\rm all}} \ln\det\Big(\delta_{i\bar{j}} - \textstyle\sum_a \Phi_{ia}\Phi^\dagger_{ja}\Big)\,. \tag{5.9}$$

The Riemann tensor is in this case not given by a tensor product of metrics, unless $p = 1$. However, its components are nevertheless related in a simple way to those of the metric in certain coordinate frames. In particular, in flat coordinates one finds the simple expression [5] [15, 16]:

$$R_{IA\,\bar{J}\bar{B}\,PC\,\bar{Q}\bar{D}} = \frac{R_{\rm all}}{2}\Big(\delta_{I\bar{J}}\,\delta_{P\bar{Q}}\,\delta_{A\bar{D}}\,\delta_{C\bar{B}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}}\,\delta_{A\bar{B}}\,\delta_{C\bar{D}}\Big)\,. \tag{5.10}$$

In this case, the map $H^{IA\,\bar{J}\bar{B}} \to R^{\bar{J}\bar{B}\,IA}{}_{PC\,\bar{Q}\bar{D}}\,H^{PC\,\bar{Q}\bar{D}}$ is singular and therefore not invertible, so that the condition (3.5) is trivially satisfied and does not give any constraint. The map $S^{IA\,JB} \to R_{PC}{}^{IA}{}_{QD}{}^{JB}\,S^{PC\,QD}$ has, on the other hand, eigenvalues $R_{\rm all}$ with degeneracy $p(p+1)(p+q)(p+q+1)/4$ and $-R_{\rm all}$ with degeneracy $p(p-1)(p+q)(p+q-1)/4$. As this map has negative eigenvalues (unless $p = 1$), the condition (3.12) is also satisfied and does not give any constraint either.

For these Kähler manifolds, nevertheless, one does find constraints by solving directly the equations (2.8). To see this, notice that we can rewrite the conditions (2.8) in a matrix form as $\mathrm{tr}(GG^\dagger) = 3$ and $R_{\rm all}\,\mathrm{tr}(GG^\dagger GG^\dagger) < 6$. Observe now that the $p \times (p+q)$ matrix G can be diagonalized by a bi-unitary transformation. More precisely, one can rewrite $G = UG^{\rm diag}V^\dagger$, where $U \in \mathrm{SU}(p)$ and $V \in \mathrm{SU}(p+q)$, and $G^{\rm diag}$ is a $p \times (p+q)$ diagonal matrix with p complex eigenvalues. Note that, using this rewriting in the two

[5] Note that the expression (5.10) can, also in this case, be rewritten in terms of the metric in flat coordinates $\delta_{IA\,\bar{J}\bar{B}} = \delta_{I\bar{J}}\delta_{A\bar{B}}$, but this decomposition takes a simple tensor product form as in (5.4) only in the maximally symmetric case corresponding to $p = 1$.

conditions (2.8) defining our problem, the matrices U and V always cancel out thanks to the cyclic property of the trace. Defining then the new p positive and real variables

$$\Theta_i = \frac{1}{\sqrt{3}} \left| \text{Eigenvalue}_i(G^{IA}) \right| , \tag{5.11}$$

one can finally rewrite (2.8) as:

$$\textstyle\sum_i \Theta_i^2 = 1 \ , \ \ \sum_i R_{\rm all}\, \Theta_i^4 < \frac{2}{3} . \tag{5.12}$$

The problem takes now exactly the same form as the one for a factorizable scalar manifold given by the product of p one-dimensional submanifolds all having the same curvature $R_i = R_{\rm all}$. The necessary condition for the existence of non-supersymmetric flat and stable vacua is then simply:

$$R_{\rm all}^{-1} > \frac{3}{2p} . \tag{5.13}$$

If the curvature satisfies the restriction (5.13), then there exist solutions, but only for a limited range of values for the variables Θ_i. More precisely, one must have $\Theta_i \in [\Theta_i^-, \Theta_i^+]$ with:

$$\begin{aligned}
\Theta_i^+ &= \begin{cases} \sqrt{\frac{1}{p} + \sqrt{\frac{2}{3}\frac{p-1}{p}\Big(R_{\rm all}^{-1} - \frac{3}{2p}\Big)}} \,, & \text{if } R_{\rm all}^{-1} < \frac{3}{2} \,, \\ 1 \,, & \text{if } R_{\rm all}^{-1} > \frac{3}{2} \,. \end{cases} \\
\Theta_i^- &= \begin{cases} \sqrt{\frac{1}{p} - \sqrt{\frac{2}{3}\frac{p-1}{p}\Big(R_{\rm all}^{-1} - \frac{3}{2p}\Big)}} \,, & \text{if } R_{\rm all}^{-1} < \frac{3}{2(p-1)} \,, \\ 0 \,, & \text{if } R_{\rm all}^{-1} > \frac{3}{2(p-1)} \,. \end{cases}
\end{aligned} \tag{5.14}$$

5.3 $\frac{\mathrm{SO}(2,q+2)}{\mathrm{SO}(2)\times\mathrm{SO}(q+2)}$

Another simple and relevant kind of Kähler coset manifolds are the Grassmanian spaces of dimension $N = q + 2$ of the form:

$$\mathcal{M} = \frac{\mathrm{SO}(2, q+2)}{\mathrm{SO}(2) \times \mathrm{SO}(q+2)} . \tag{5.15}$$

This coset manifold can be parametrized by a vector of complex fields ϕ_i, with $i = 1, 2, \ldots, q+2$, and a Kähler potential given by:

$$K = -\frac{2}{R_{\rm all}} \ln\Big(1 - 2\textstyle\sum_i \Phi_i \Phi_i^\dagger + \sum_{i,j} (\Phi_i \Phi_j^\dagger)^2\Big) . \tag{5.16}$$

The Riemann tensor in flat coordinates is in this case found to have the slightly less trivial form [15]:

$$R_{I\bar{J}P\bar{Q}} = \frac{R_{\rm all}}{2}\Big(\delta_{I\bar{J}}\,\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}} - \delta_{IP}\,\delta_{\bar{J}\bar{Q}}\Big) . \tag{5.17}$$

In this case, the map $H^{I\bar{J}} \to R^{\bar{J}I}{}_{P\bar{Q}}\, H^{P\bar{Q}}$ is singular so that the condition (3.5) is satisfied and does not give any constraint. The map $S^{IJ} \to R_P{}^I{}_Q{}^J\, S^{PQ}$ has, on the other hand, eigenvalues $R_{\rm all}$ with degeneracy $(q+1)(q+3)/2$ and $-\,q\,R_{\rm all}/2$ with degeneracy 1. The condition (3.12) is therefore also satisfied and does not give any constraint either.

Nevertheless in this case one can also solve directly the conditions (2.8) in an exact way. Using a vector notation, one can rewrite the conditions (2.8) as $G\cdot G^* = 3$ and $2(G\cdot G^*)^2 - (G\cdot G)(G^*\cdot G^*) < 12/R_{\rm all}$. However now the two conditions depend only on two independent combinations of the variables. Indeed, introducing the two real positive variables

$$\Theta_{1,2} = \frac{1}{\sqrt{6}}\sqrt{\sum_I |G^I|^2 \pm \sqrt{\left(\sum_I |G^I|^2\right)^2 - \left|\sum_I (G^I)^2\right|^2}}\,, \tag{5.18}$$

the two conditions can be rewritten as:

$$\Theta_1^2 + \Theta_2^2 = 1\ ,\ \ R_{\rm all}\left(\Theta_1^4 + \Theta_2^4\right) < \frac{2}{3}\,. \tag{5.19}$$

The problem has now the same form as the one for a factorizable scalar manifold given by the product of two one-dimensional submanifolds with the same curvature $R_i = R_{\rm all}$. The necessary condition for the existence of non-supersymmetric flat and stable vacua is then:

$$R_{\rm all}^{-1} > \frac{3}{4}\,. \tag{5.20}$$

If the curvature satisfies the restriction (5.20), then there exist solutions, but only for a limited range of values for the variables $\Theta_{1,2}$. More precisely, one must have $\Theta_{1,2} \in [\Theta_{1,2}^-, \Theta_{1,2}^+]$ with:

$$\begin{aligned}
\Theta_{1,2}^+ &= \begin{cases} \sqrt{\dfrac{1}{2} + \sqrt{\dfrac{1}{3}\left(R_{\rm all}^{-1} - \dfrac{3}{4}\right)}}\,, & \text{if } R_{\rm all}^{-1} < \dfrac{3}{2}\,, \\ 1\,, & \text{if } R_{\rm all}^{-1} > \dfrac{3}{2}\,. \end{cases} \\
\Theta_{1,2}^- &= \begin{cases} \sqrt{\dfrac{1}{2} - \sqrt{\dfrac{1}{3}\left(R_{\rm all}^{-1} - \dfrac{3}{4}\right)}}\,, & \text{if } R_{\rm all}^{-1} < \dfrac{3}{2}\,, \\ 0\,, & \text{if } R_{\rm all}^{-1} > \dfrac{3}{2}\,. \end{cases}
\end{aligned} \tag{5.21}$$

6. Moduli spaces in string models

Many of the scalar manifolds arising in the moduli sector of string compactifications fall into the classes of factorizable or symmetric spaces, actually homogeneous coset spaces, that we have studied in the previous sections. These sectors include the neutral fields controlling the size of the coupling and the geometry of the compactification manifold, as well as possible Wilson lines for the hidden gauge group. They represent natural candidates for the hidden sector in this type of models, and it is therefore of evident interest to apply to these cases

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our results on the conditions under which flat and locally stable non-supersymmetric vacua can exist.

In the simplest case of orbifold and orientifold compactifications, the untwisted sector moduli space must be a subgroup of the moduli space emerging for maximally supersymmetric toroidal reductions, which is uniquely fixed by the fact that there are 6 extra internal dimensions and by the rank s of the hidden gauge group. More precisely, at leading order one finds:

$$\mathcal{M} \subset \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(6,6+s)}{\mathrm{SO}(6)\times\mathrm{SO}(6+s)}\,. \tag{6.1}$$

The first factor is always present and is associated to the universal dilaton modulus S controlling the coupling. The second factor is instead broken by the orbifold or orientifold projection to a subgroup that has the form of a product of coset Kähler manifolds of the types studied in section 5. It is associated to the Kähler and complex structure moduli T and U controlling the size and the shape of the compactification manifold, and the Wilson lines X of the hidden gauge group.

The simplest situation that can appear for a given modulus Φ_i is described by a Kähler potential of the form [17]:

$$K = -\,n_i \ln\Big(\Phi_i + \Phi_i^\dagger\Big)\,. \tag{6.2}$$

It is straightforward to show that this potential can be written in the form (5.3), with $R_{\rm all} = 2/n_i$, by means of a holomorphic change of variables and a Kähler transformation. The corresponding Kähler manifold is therefore

$$\mathcal{M} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}\,. \tag{6.3}$$

In this simplest case, the scalar manifold is both one-dimensional and symmetric, and for the flatness and stability conditions this corresponds to having one field with curvature $R_i = 2/n_i$. In the presence of several fields with Kähler potentials of the form (6.2), the flatness and stability conditions imply $\sum_k R_k^{-1} > 3/2$, which requires that $\sum_k n_k > 3$, as found in ref. [9].

A first relevant generalization involves the Kähler moduli controlling the size of some cycle in the internal manifold, and is due to the possible presence of Wilson lines around that cycle. Each modulus T_i can in principle mix with an arbitrary number q_i of Wilson lines X_{a_i}, with $a_i = 1, 2, \ldots, q_i$. The simple one-dimensional space (6.3) is then enhanced to a (q_i+1)-dimensional space with a Kähler potential given by [18]:

$$K = -\,n_i \ln\Big(T_i + T_i^\dagger - \textstyle\sum_{a_i} X_{a_i} X_{a_i}^\dagger\Big)\,. \tag{6.4}$$

This Kähler potential (6.4) can be written in the form (5.3), with $R_{\rm all} = 2/n_i$, by means of a holomorphic field redefinition and a Kähler transformation. The corresponding Kähler manifold is therefore isomorphic to a maximally symmetric coset space of the type studied in subsection 5.1:

$$\mathcal{M} = \frac{\mathrm{SU}(1,q_i+1)}{\mathrm{U}(1)\times\mathrm{SU}(q_i+1)}\,. \tag{6.5}$$

Recalling the results of subsection 5.1, we conclude that, as far as the flatness and stability conditions are concerned, this situation is equivalent to having a single field with curvature $R_i = 2/n_i$. In the presence of several groups of fields with Kähler potentials of the form (6.4), the flatness and stability conditions imply then that $\sum_k R_k^{-1} > 3/2$, which requires again that $\sum_k n_k > 3$. Wilson lines do therefore not change qualitatively the situation. Their presence enhances the minimal geometry in such a symmetric way that the only effect of the corresponding auxiliary fields is to contribute together with the involved modulus to the combination of auxiliary fields (5.5) that is relevant to find the constraints.

Another interesting and relevant generalization can appear for Kähler moduli in particularly symmetric models, like $\mathbf{Z}_2$ or $\mathbf{Z}_3$ orbifolds, and is due to the presence of additional non-standard moduli of this type. More precisely, a set of p_r Kähler moduli T_{i_r} with equal parameter n_r, where $i_r = 1, 2, \ldots, p_r$, can mix with $p_r(p_r - 1)$ extra Kähler moduli T_{α_r}, where $\alpha_r = 1, 2, \ldots, p_r(p_r - 1)$. There are then in total p_r^2 Kähler moduli, which can be organized in a matrix $T_{i_r j_r}$, where $i_r, j_r = 1, 2, \ldots, p_r$. The scalar manifold associated to the original p_r moduli, which is a product of p_r copies of the minimal space (6.3), is then enhanced to a p_r^2-dimensional space with a Kähler potential given by [19]:

$$K = -\, n_r \ln \det\Big(T_{i_r j_r} + T^\dagger_{i_r j_r}\Big)\,. \tag{6.6}$$

One can use a holomorphic field redefinition and a Kähler transformation to rewrite this Kähler potential in the form (5.9), with $R_{\rm all} = 2/n_r$, and the corresponding scalar manifold is therefore a particular complex Grassmanian manifold of the type studied in subsection 5.2:

$$\mathcal{M} = \frac{\mathrm{SU}(p_r, p_r)}{\mathrm{U}(1) \times \mathrm{SU}(p_r) \times \mathrm{SU}(p_r)}\,. \tag{6.7}$$

From the analysis developed in subsection 5.2 we can then conclude that the flatness and stability conditions depend only on p_r independent combinations of fields (5.11) with identical curvatures $R_r = 2/n_r$. In the presence of several groups of fields with Kähler potentials of the form (6.6), the flatness and stability conditions imply that $\sum_r p_r\, R_r^{-1} > 3/2$, which reduces to the condition $\sum_r p_r\, n_r > 3$. The extra off-diagonal Kähler moduli are therefore qualitatively irrelevant for the restrictions imposed on the curvature, and they just combine with the diagonal Kähler moduli into the combinations of fields (5.11) relevant to find the constraints.

The two deformations that we have considered so far, related to the presence of extra Wilson lines and off-diagonal Kähler moduli, can also occur simultaneously. In this more general situation, a set of p_r Kähler moduli T_{i_r} with equal parameter n_r, where $i_r = 1, 2, \ldots, p_r$, can mix with $p_r(p_r-1)$ extra Kähler moduli T_{α_r}, where $\alpha_r = 1, 2, \ldots, p_r(p_r-1)$, as well as $p_r q_r$ Wilson lines $X_{i_r a_r}$, where $a_r = 1, 2, \ldots, q_r$. There are then p_r^2 Kähler moduli, which can be organized in a matrix $T_{i_r j_r}$, and in addition $p_r q_r$ Wilson lines $X_{i_r a_r}$. The scalar manifold associated to the original p_r moduli, which is a product of p_r copies of the minimal space (6.3), is then enhanced to a $p_r(p_r + q_r)$-dimensional space with a Kähler potential given by [19]:

$$K = -\, n_r \ln \det\Big(T_{i_r j_r} + T^\dagger_{i_r j_r} - \textstyle\sum_{a_r} X_{i_r a_r} X^\dagger_{j_r a_k}\Big)\,. \tag{6.8}$$

This can be shown to be equivalent to a Kähler potential of the form (5.9) with $R_{\rm all} = 2/n_r$, and the corresponding Kähler manifold is now the general case of the complex Grassmanian manifolds studied in subsection 5.2:

$$\mathcal{M} = \frac{\mathrm{SU}(p_r, p_r + q_r)}{\mathrm{U}(1) \times \mathrm{SU}(p_r) \times \mathrm{SU}(p_r + q_r)} \,. \tag{6.9}$$

As in the previous case we can use the information given in subsection 5.2 to conclude that the flatness and stability conditions depend only on p_r independent combinations of fields (5.11) with identical curvatures $R_r = 2/n_r$. In the presence of several groups of fields with Kähler potentials of the form (6.8), the flatness and stability conditions imply that $\sum_r p_r R_r^{-1} > 3/2$, which requires as in the previous cases that $\sum_r p_r\, n_r > 3$. This means that neither the extra off-diagonal Kähler moduli nor the extra matter fields $X_{i_r a_r}$ are relevant for the restrictions imposed on the curvature, and they just combine with the diagonal Kähler moduli into the combinations of fields (5.11) relevant to find the constraints.

There is yet another type of interesting enhancement that can appear for Kähler and complex structure moduli in certain specific models, like $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifolds, and which is due to Wilson lines mixing with these two kinds of fields simultaneously. More precisely, a pair of two Kähler and complex structure moduli T_r and U_r associated to the same submanifold and having the same parameter n_r can mix with a number q_r of Wilson lines X_{a_r}, with $a_r = 1, 2, \ldots, q_r$, associated to that submanifold. The scalar manifold associated to the two original moduli, which is a product of two copies of the minimal space (6.3), is then enhanced to a $(q_r + 2)$-dimensional space with a Kähler potential given by [20]:

$$K = -\, n_r \ln \left((T_r + T_r^\dagger)(U_r + U_r^\dagger) - \textstyle\sum_{a_r} (Z_{a_r} + Z_{a_r}^\dagger)^2 \right) . \tag{6.10}$$

In this case, the scalar manifold can be shown to be

$$\mathcal{M} = \frac{\mathrm{SO}(2, 2 + q_r)}{\mathrm{SO}(2) \times \mathrm{SO}(2 + q_r)} \,. \tag{6.11}$$

This is a Grassmanian coset of the type described in subsection 5.3. We can then conclude that the flatness and stability conditions depend only on the two independent combinations of fields (5.18) with identical curvatures $R_r = 2/n_r$. In the presence of several such pairs of fields with Kähler potentials of the form (6.10), the flatness and stability conditions imply $\sum_r 2\, R_r^{-1} > 3/2$, which requires that $\sum_r 2\, n_r > 3$. The presence of the Wilson lines is once again qualitatively irrelevant, and just changes the relevant combinations (5.18) of fields.

Summarizing we have learned that all the possible enhancements we have considered of the minimal factorized moduli space given by a product of factors (6.3) do not qualitatively change the form of the flatness and stability conditions. Due to the very high degree of symmetry of these enhancements, the only net effect of the extra fields involved is to change the combinations of fields that are relevant for the conditions. In particular, they do not change neither the number of relevant combinations nor the values of the associated curvatures. The curvature constraints for the existence of flat and stable non-supersymmetric vacua then depend only on the number N of diagonal moduli Φ_i and their

associated numerical parameter n_i. The problem is thus identical to the one arising in a N-dimensional factorizable space with constant curvatures given by $R_i = 2/n_i$. In all the situations analyzed here, the existence of non-supersymmetric flat and stable vacua is then permitted only if the parameters n_i fulfill the condition

$$\sum_k n_k > 3\,. \tag{6.12}$$

The only peculiarity of situations where the moduli space is enhanced is that some of the n_i's have the same values.

In the simplest situations arising in string compactifications, there are seven moduli fields. These are just the S field, the three Kähler moduli T_i and the three complex structure moduli U_i which in the simplest situations have diagonal potential of the form (6.2), or more in general we can have mixtures of these fields with the extra moduli fields enhancing the Kähler manifold spanned by them. Each of these fields has $n_i = 1$, and this means that none of them can dominate on its own supersymmetry breaking, as was already pointed out in ref. [9]. In fact, as can be easily seen from (4.5), the variables Θ_i have an upper and a lower bound given by:

$$\Theta_i^{\pm} = \sqrt{\frac{n_i \pm \sqrt{n_i\big(\sum_k n_k - n_i\big)\big(\sum_k n_k - 3\big)}}{\sum_k n_k}}\,. \tag{6.13}$$

This constrains as well the values that the F auxiliary fields associated to each of the independent combinations of fields can take, which depend both on the diagonal and the off-diagonal moduli that might be present.

7. Radion in Randall-Sundrum models

The general results we have derived in the previous sections have also interesting implications for phenomenological models with a single extra dimension, like for instance supersymmetric Randall-Sundrum models with generic warping k [21]. In such models, the classical effective Kähler potential has a form that is constrained by locality and general covariance. Denoting by M_5 the 5-dimensional Planck scale, by T the radion chiral multiplet controlling the size of the extra dimension, and by X_a, $a = 1, 2, \ldots, q$, and $\tilde{X}_{\tilde{a}}$, $a = 1, 2, \ldots, \tilde{q}$, the matter fields at the two branes, one finds that [22, 23]:

$$K = -3\ln\left[\frac{M_5^3}{k}\Big(1 - e^{-k(T+T^\dagger)}\Big) - \frac{1}{3}\sum\nolimits_a X_a^\dagger X_a - \frac{1}{3}\sum\nolimits_{\tilde{a}} \tilde{X}_{\tilde{a}}^\dagger \tilde{X}_{\tilde{a}}\, e^{-k(T+T^\dagger)}\right]. \tag{7.1}$$

It is straightforward to show, by means of a simple rescaling of the fields, that this describes a $(q + \tilde{q} + 1)$-dimensional maximally symmetric coset space of the form:

$$\mathcal{M} = \frac{\mathrm{SU}(1, q + \tilde{q} + 1)}{\mathrm{U}(1) \times \mathrm{SU}(q + \tilde{q} + 1)}\,. \tag{7.2}$$

From (7.1) we can read that the curvature is given by $R_{\mathrm{all}} = 2/3$, and therefore, using the result given in (5.7), it marginally violates the curvature bound allowing for the existence

of flat and stable non-supersymmetric vacua. This means in particular that corrections to the Kähler potential (7.1) will be crucial, since even a slight change on the curvature can allow it to fulfill the necessary condition (5.7).

This fact has strong implications on models of radius stabilization within this setup. For instance in the model proposed in ref. [24, 23] the radion superpotential induced by some gaugino condensation in the bulk is used to stabilize the radion at an AdS point, which is then uplifted to a Minkowski vacuum thanks to a brane sector. Using our previous result, we conclude that this model cannot work at leading order if the brane sector sector has a Kähler potential as in (7.1). To improve the situation one needs to have some non-linearity in the matter sector (this would reduce the high degree of symmetry of (7.2)), like for instance a non-trivial field-dependent wave-function factor. This was already suggested in ref. [25], where it was assumed that such a wave-function would stabilize the brane scalar fields at vanishing values. The results derived here show that this is actually mandatory in order for this model to work.

In general, quantum effects induce non-trivial corrections to the tree-level Kähler potential (7.1). These corrections can be either divergent local effects that can be reabsorbed by a renormalization of the parameters in (7.1), or finite non-local effects that induce, on the contrary, a correction with a different dependence on the fields (see for instance refs. [25, 26]). These Casimir-like corrections modify the structure of (7.1), and can therefore be potentially useful to lower the effective curvature below the critical value $R_{\rm all} = 2/3$ obtained at the classical level.

8. Conclusions

In this paper, we have analyzed in more generality the implications of the flatness and stability constraints derived in ref. [9] for supergravity theories where only chiral multiplets are relevant for supersymmetry breaking. We have explored in detail special cases where the Kähler manifolds spanned by the scalar fields are such that it is possible to work out in full generality the implications of these constraints. We have studied in particular the coset Kähler manifolds that are relevant for the moduli sector of string models. Since these are homogeneous spaces with constant curvature, the implications of the constraints are in this case particularly simple and directly related to the parameters of the theory. We have found that the conditions for the existence of flat and stable non-supersymmetric vacua impose in these cases strong constraints on the Kähler geometry and also on the values that the auxiliary fields can take (as was also the case for the examples considered in [9]). We were able to show that the basic symmetry enhancements due to the addition of extra off-diagonal and/or untwisted matter fields that extend the minimal space of products of $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$ factors to more complicated coset manifolds are qualitatively irrelevant as far as the constraints on the Kähler geometry are concerned. Actually, the additional fields were found to change only the combinations of auxiliary fields that are relevant for the constraints, leaving their number and the associated constraints unchanged. We have also explored the case of completely arbitrary scalar manifolds, for which the original variational problem defined by the constraints is not exactly solvable. Nevertheless, we were able to

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derive explicit results also in this more general case by further simplifying the conditions defining the problem in a way that made the new variational problem exactly solvable. In this way, we were able to obtain weaker but completely general necessary conditions.

There are many avenues of future work following these lines. One of the most interesting is to perform the the same kind of analysis as the one presented here when also vector multiplets participate to supersymmetry breaking. The presence of vector multiplets with significant D auxiliary fields, in addition to chiral multiplets with non-vanishing F auxiliary fields, can alleviate the restrictions found in ref. [9] and in this paper (where D-terms where neglected with respect to F-terms) [6]. More precisely, some of the vector multiplets can gauge some isometries of the chiral multiplet sector, and the corresponding D-terms are then related to the F-terms through the Killing potentials specifying the gauging. The progress made in the present paper concerning symmetric spaces should be relevant to study this interesting but more complicated situation more efficiently.

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